Addressing

The underlying abstraction is a byte stream addressed by positions from the set:

$$\mathcal{P} = \{p \mid 0 \leq p \leq \texttt{size}\} \subset \mathbb{N}_0$$

In slight abuse of notation we define for $s, e \in \mathcal{P}$

$$[s, e) = \begin{cases} \{p \in \mathcal{P} \mid s \leq p < e\} & \text{if } s < e \\ s & \text{if } s = e \\ \emptyset & \text{otherwise} \end{cases}$$

Resulting in the set of ranges

$$\mathcal{R} = \{[s, e) \mid s, e \in \mathcal{P}, s < e\}$$

Selections

Selections are non-empty, directed ranges

$$\mathcal{S} = \mathcal{R} \times \{\leftarrow, \downarrow, \rightarrow\}$$

Forming a totally ordered set $\langle \mathcal{S}, \leq \rangle$, according to:

$$([s_1, e_1), d_1) \leq ([s_2, e_2), d_2) \iff s_1 < s_2 \vee (s_1 = s_2 \wedge e_1 \leq e_2)$$

An invalid selection is denoted by $(\emptyset, \cdot)$

Selections: Membership, Length and Direction

A selection $s = (r, d) \in \mathcal{S}$

- contains $p \in \mathcal{P} \iff p \in r$.
- has length $|s| = |r|$
- is anchored and left extending if $d = \leftarrow$
- is non-anchored or following if $d = \downarrow$, this implies $|s| = 1$
- is anchored and right extending if $d = \rightarrow$
- can be flipped

$$\text{flip}(s) = \begin{cases} (r, \rightarrow) & \text{if } d = \leftarrow \\ s & \text{if } d = \downarrow \\ (r, \leftarrow) & \text{if } d = \rightarrow \end{cases}$$

Selections: Cursor and Anchor

Each selection has 2 end points, cursor and anchor, which are *over* a character.

$$
\begin{aligned}
&\mathcal{S} \to \mathcal{P} \\
\text{cursor}\colon ([s, e), d) &\mapsto \begin{cases} e - 1 & \text{if } d = \rightarrow \\ s & \text{otherwise} \end{cases} \\
\text{anchor}\colon ([s, e), d) &\mapsto \begin{cases} e - 1 & \text{if } d = \leftarrow \\ s & \text{otherwise} \end{cases}
\end{aligned}
$$

For singleton selections both cursor and anchor are on the same character.

Selections: Motions

- Normal mode

  Non-anchored singleton selections $([p, p+1), \downarrow)$ for cursor at $p \in \mathcal{P}$.

- Visual mode

  Selections are anchored, only the cursor is adjusted, the anchor remains fixed.

Selections: Cover and Normalization

The cover of $S \subseteq \mathcal{S}$ is

$$\text{cover}\colon 2^{\mathcal{S}} \to 2^{\mathcal{P}}$$
$$\text{cover}(S) = \{p \in \mathcal{P} \mid (r, d) \in S, p \in r\}$$

We define a normalization operator on a set of selections:

$$\text{normalize}\colon 2^{\mathcal{S}} \to 2^{\mathcal{S}}$$

such that:

1. $(\emptyset, \cdot) \notin \text{normalize}(S)$
2. $\text{cover}(\text{normalize}(S)) = \text{cover}(S)$
3. $\forall s_1 = (r_1, d_1), s_2 = (r_2, d_2) \in S : s_1 \neq s_2 \implies r_1 \cap r_2 = \emptyset$
4. $\forall s_1 = (r_1, d_1) \in S :$
   $(\forall s_2 = (r_2, d_2) \in S \setminus \{s_1\} : r_1 \cap r_2 = \emptyset) \implies s_1 \in \text{normalize}(S)$

A normalized selection set is an interval order.

Selections: Intersect and Union

Operations on ordered sets of selections, $S_1, S_2 \subseteq \mathcal{S}$

- intersect

$$S_1 \cap S_2 = \{(s_1 \cap s_2, d_1) \mid (s_1, d_1) \in S_1, (s_2, d_2) \in S_2\}$$

  note: not symmetric

- union

$$S_1 \cup S_2 = \text{normalize}(S_1 \cup S_2)$$

Selections: Complement and Minus

- complement
- minus

These behave as "expected".

TODO: define them formally

Selections: Pairwise Merge

Pairwise merge with $\mathrm{f} : \mathcal{S} \times \mathcal{S} \to \mathcal{S}$

$$\mathrm{merge}(S_1, S_2) = \{\mathrm{f}(u_i, v_i) \mid u_1 < \cdots < u_n \in S_1, v_1 < \cdots < v_m \in S_2\}$$

i.e. start with the minimum and repeatedly combine the immediate successor of both sets.

- $\mathrm{union}(([s_1, e_1), d_1), ([s_2, e_2), d_2)) = ([\min(s_1, s_2), \max(e_1, e_2)), d_1)$
- $\mathrm{intersect}(([s_1, e_1), d_1), ([s_2, e_2), d_2)) = ([\max(s_1, s_2), \min(e_1, e_2)), d_1)$
- $\mathrm{left}(s_1, s_2) = \min(s_1, s_2)$
- $\mathrm{right}(s_1, s_2) = \max(s_1, s_2)$
- $\mathrm{longer}(s_1, s_2) = \begin{cases} s_1 & \text{if } |s_1| > |s_2| \\ s_2 & \text{otherwise} \end{cases}$
- $\mathrm{shorter}(s_1, s_2) = \begin{cases} s_1 & \text{if } |s_1| < |s_2| \\ s_2 & \text{otherwise} \end{cases}$